

MLE FOR VAR-GAMMA DISTRIBUTION

Assume the data pairs $(y_1, z_1), \dots, (y_n, z_n)$ are an i.i.d. sample from the distribution with density

$$f(y, z, \theta, \sigma) = e^{-y} \cdot \frac{1}{\sqrt{2\pi y \sigma}} e^{-\frac{(z-\theta y)^2}{2y\sigma^2}}$$

for $y > 0$ and $\sigma > 0$.

- a. Find the maximum likelihood estimators of θ and σ^2 . Are the estimators unbiased?
- b. Find the exact standard errors of $\hat{\theta}$ and $\hat{\sigma}^2$.
- c. Compute the Fisher information matrix.
- d. Find the standard errors of the maximum likelihood estimators using the Fisher information matrix. Comment on your findings.
- e. Generate samples by computer and estimate the parameters. Repeat the procedure $N = 1000$ times and compute the empirical standard error. Comment.